ADDITIONAL MATHEMATICS

Paper 0606/01
Paper 1

General comments

Most candidates were able to make a reasonable attempt at the paper. There was a wide range of marks. Some candidates scored highly while others obtained scores in single figures and were clearly unprepared for the paper. The first few questions on the paper proved to be fairly straightforward to most candidates and resulted in many candidates making a sound start to the paper. Later questions proved to be of a more demanding nature.

Candidates should be reminded of the necessity of showing all working clearly and in appropriate detail.

Comments on specific questions

Question 1

Candidates had been generally well prepared for this question as most were able to gain some marks if not all.

(i) Most candidates used $s = r\theta$, working with radians throughout, but a few chose to work in degrees and then convert to radians. A few candidates misunderstood the question and thought they were dealing with a triangle rather than a sector of a circle. This misunderstanding was then carried through to part **(ii)**.

(ii) Most candidates were able to gain marks by applying a correct method to find the area of the sector required with the exception as referred to above.

Answers: **(i)** 0.8 radians; **(ii)** 90 cm^2.

Question 2

Many candidates were able to gain full marks, showing a good understanding of the syllabus content for this topic. Simple common errors included the misconceptions that 'crossing the *x*-axis' meant that $x = 0$, $\sqrt[3]{8} = \pm 2$ and $\frac{\mathrm{d}}{\mathrm{d}x}\left(x^3\right) = 2x^2$. Most candidates were able to find the equation of a straight line using a numerical value for the gradient of the normal.

Answer: $y = -\frac{1}{12}x + \frac{1}{6}$.

Question 3

Very few candidates were unable to gain some marks for this question. Most candidates were able to apply appropriate trigonometric identities and simplify to obtain the required results. However, some examples of inappropriate algebra and simplification meant that some were unable to proceed correctly with the proof.

Question 4

Most candidates were able to make an attempt at this question and gain some credit. The most common solutions involved equating the equation of the curve to that of the line and using '$b^2 = 4ac$' on the resulting quadratic equation. A minority of candidates chose to equate the gradient of the curve with that of the line. Of these, many could not get beyond the fact that '$xk = 4$'. Those that did use this method correctly found it a quick way to obtain the solution required for part **(ii)**.

Answers: **(i)** 2; **(ii)** (2, 7).

Question 5

(i) The great majority of candidates were able to gain marks by attempting to express the equation in terms of powers of 3. Occasional algebraic slips sometimes lost a candidate a mark. A few candidates were able to obtain a correct solution by using logarithms.

(ii) This was not answered as well as part **(i)**, errors being often due to dealing with negative indices within a fraction.

Answers: **(i)** 2; **(ii)** –2, 1.

Question 6

The great majority of candidates were able to make a good attempt at this question by applying a correct method of solution and so many were able to obtain full marks. Occasionally candidates left their answer in terms of factors, but in this case they were penalised by the loss of only one mark if the factors were correct.

Less well prepared candidates attempted spurious methods of solving which rarely resulted in any marks.

Answer: –5, 0.5, 3.

Question 7

(i) The majority of candidates failed to recognise that xe^{3x} needed to be differentiated as a product, but many were able to gain credit for the correct differentiation of $-e^{3x}$. However, too many candidates appeared to be unfamiliar with the differentiation of exponential functions which meant that they were unable to attempt either part of this question with any success.

(ii) Of the candidates who got part **(i)** correct a high proportion went on and were able to gain full marks for part **(ii)** by making use of their answer to part **(i)**. There were still candidates who failed to realise the implication of the word 'Hence' used on the question paper and tried various incorrect attempts at integration. Those candidates that used integration by parts and obtained a correct solution were given full credit even though this method is not a syllabus requirement.

Answers: **(i)** $3xe^{3x}$; **(ii)** $\frac{1}{3}\left(xe^{3x} - \frac{e^{3x}}{3}\right)$.

Question 8

(i) Most candidates were able to differentiate correctly, with a few candidates using an incorrect formula. Most realised that they had to equate $\frac{dy}{dx}$ to zero but proceeded with varying amounts of success. Some dubious methods of solution resulted in no marks, whilst many candidates failed to give both the required solutions of the equation $x^2 = 9$.

(ii) Provided the differentiation to part **(i)** was correct, most candidates were able to gain full marks for this part even though some incorrect notation in the form of small changes was sometimes used. Those that did not have a correct differentiation from part **(i)** were usually able to gain some credit for correct methods used.

Answers: **(i)** –3, 3; **(ii)** 0.32 units per second.

Question 9

This question was not done at all well by many candidates. Many were clearly unaware of the difference between position vectors and distance, likewise between velocity vectors and speed. Position and velocity vectors were frequently added or subtracted without realisation of correct methods to employ.

(i) A common incorrect solution was $-40\sqrt{2}\ \mathbf{i}\ +80\sqrt{2}\,\mathbf{j}$, a combination of a position vector with a speed.

(ii) This was often better understood, but often lacked accuracy due to an incorrect velocity vector in part **(i)**.

(iii) This was often better understood, but often lacked accuracy due to an incorrect velocity vector in part **(i)**.

(iv) Again velocity vectors and distance were used incorrectly for the two ships, and many candidates did not use the fact that the ships meet when their position vectors are the same. This meant that most were unable to gain marks in this part.

Answers: **(i)** 10**i** + 10**j**; **(ii)** 16**i** + 28**j**; **(iii)** 2**i** + 4**j**; **(iv)** 13 30 hours at 31**i** + 43**j**.

Question 10

Many candidates scored full marks on this question and most of these candidates set out their solutions clearly. There was, however, a tendency amongst a small number of candidates to omit essential working and to give equations of lines and coordinates of points without supporting evidence.

(i) The most common errors, apart from numerical errors, were to misquote the result $m_1m_2 = -1$ for the gradients of perpendicular lines and/or to assume mistakenly that the point *C* was the mid-point of either the line *AB* and/or the line *PQ*.

(ii) A correct method for finding the required area was usually employed.

Answers: **(i)** (4, 6), (8.5, 0); **(ii)** 37.5.

Question 11

Most candidates attempted this question to some extent. Many failed to realise that 'a suitable straight line graph', meant that they had to re-write the given relationship in a logarithmic form. Again, many just plotted *s* against *t* and formed a curve or joined two of their points to form a straight line. Those that made no attempt at logarithms were only able to gain marks for part **(iii)**.

(i) A significant number of candidates were able to identify the correct logarithmic form of the equation but then failed to plot the appropriate straight line graph.

(ii) Some candidates chose to calculate the values of *k* and of *n* algebraically; unfortunately these candidates gained no credit as they were required to make use of their graph, thus testing a specific syllabus requirement. Others penalised themselves by not drawing a complete horizontal axis and thus misinterpreted their intercept on the vertical axis. Reversal of the axes was rarely noticed by candidates and thus incorrect values for *n* and *k* were obtained. A number of candidates failed to use points that lay on their straight line when calculating the gradient, choosing instead to use values from their tables, thus penalising themselves. Many did use appropriate values that lay on their straight line but failed to deal with the negative aspect of the gradient.

(iii) Most candidates who had plotted any line or curve were able to obtain at least a method mark for this part as they had been asked to make an estimate. Those candidates that had produced a curve very often failed to draw it accurately enough to gain full marks.

Answers: **(ii)** 7900 to 10 000, –1.4 to –1.0; **(iii)** 72 to 92.

Question 12 EITHER

(i) Most candidates were able to obtain the correct amplitude.

(ii) Many candidates failed to obtain the correct period as they divided the period of sin x by 3 instead of multiplying by 3.

(iii) Most candidates were able to make a correct attempt at solving the obtained trigonometric equation and most were able to obtain the value of x at the point A. Common errors were often in the finding of the value of x at the point B and finding the solutions in terms of degrees rather than radians. The latter was not penalised at this point, but candidates who continued to work with degrees in part **(iv)** were penalised in part **(iv)**.

(iv) The most common causes for the loss of marks for candidates were using limits in terms of degrees, incorrect integration of $\sin\left(\frac{x}{3}\right)$ and failing to take into account the necessary subtraction of the area of a rectangle. Very few candidates retained their working in terms of π and $\sqrt{3}$ throughout the necessary calculations, resulting in many cases of premature approximation.

Answers: **(i)** 1; **(ii)** 6π; **(iii)** $\frac{\pi}{2}, \frac{5\pi}{2}$; **(iv)** 2.05.

Question 12 OR

(i) Correct answers were rarely seen as most candidates chose to give their value of t in degrees rather than correctly in radians.

(ii) Whilst most candidates realised that they had to differentiate to obtain an expression for the acceleration, there was poor differentiation which also affected later parts of the question.

(iii) Most candidates were able to make appropriate substitutions but were unable to gain full credit due to previous poor differentiation.

(iv) Completely correct sketches were rare, although credit was given for those candidates who knew how to deal with their value for k and the amplitude of the curve.

(v) As in part **(ii)**, poor applications of calculus techniques, this time involving integration were all too common, so completely correct solutions were rare even though most candidates realised that they needed to integrate to obtain an expression for displacement.

Answers: **(i)** $\frac{\pi}{8}$ or 0.392; **(ii)** $-4k \sin 4t$; **(iii)** 3; **(v)** 0.375.

ADDITIONAL MATHEMATICS

Paper 0606/02

Paper 2

General comments

There was a wide range of marks. All candidates appeared to have sufficient time to attempt all questions.

Question 4(i) was generally well done as were **Questions 5(i)**, **7(iii)**, **7(iv)**, **8(ii)**, **9(i)**, **9(ii)**, **10** and **11**.

Question 4(ii) was less well done as were **Questions 6**, **8(i)**, **12 EITHER** and **12 OR**.

Comments on specific questions

Question 1

Most candidates were able collect marks on this question although it was surprising how many candidates who scored high marks overall lost their marks on this question. In part **(a)** all but the weakest knew what to do although a few wrote 'n($X \cup Y$)'. Those that introduced the universal set symbol often applied it incorrectly.

In the Venn diagram candidates frequently failed to include all of the data as needed. They generally knew how to determine the data because they usually obtained the right answers for parts **(b)(i)** and **(ii)**. In part **(b)(iii)** a few failed to add the two values 7 and 4.

Answers: **(a)** $(X \cup Y)'$ or $X' \cap Y'$; **(b)(ii)** 9, **(iii)** 11.

Question 2

The inverse matrix was generally correct and a higher proportion of candidates used it to solve the equations than previously. There was still a significant number though who either post-multiplied or solved algebraically – often arriving at the correct values.

Answers: $\frac{1}{10}\begin{pmatrix} 4 & -6 \\ -3 & 7 \end{pmatrix}$; $x = 5$, $y = -3$.

Question 3

Most candidates were aware of the modulus idea but did not use a sufficiently big domain. The usual mistake was for the curve not to go beyond 6 but a many others stopped short of this. Some candidates showed so little that it was unclear whether it was the sketch of a modulus function.

There was sometimes page after page of meaningless or irrelevant calculations before any attempt at the sketch.

Question 4

Candidates found part **(i)** to be quite straightforward. As anticipated a small number of candidates failed to use 2 raised to the power 4 and arrived at $15 \times 2 = 30$ as their coefficient. Most candidates isolated the 240 but a number listed the whole series.

There were more problems with part **(ii)** although again it was often well answered. Those that failed to use the powers of 2 correctly in part **(i)** invariably made the same error in part **(ii)**. The most common failing was to only use the first four terms of the expansion and to take $160x^3$ and multiply that term alone by $\left(1-\frac{x}{4}\right)$. There were also occasional errors with the signs usually leading to 280 rather than 200 as the required coefficient.

Answers: **(i)** 240; **(ii)** 200.

Question 5

The differentiation was generally successful in part **(i)**, but errors sometimes occurred in the second term, $\frac{96}{x^2}$ being the most common slip-up. However, in part **(ii)** candidates frequently failed to read the phrase 'Use your expression...' and instead chose to find the exact increase in y as x increased from 2 to 2.04. There were also several incidents of misreading 0.04 as 0.4.

Answers: **(i)** $12x-\frac{96}{x^4}$; **(ii)** 0.72.

Question 6

All three parts were answered with varying success.

As with previous papers, candidates frequently had difficulty with finding the range of f. Many simply found f(3) and stated that the range was $y = 2$. A small number used the wrong inequality sign, i.e. > rather than ›.

In part **(ii)** there were some poor attempts to make x the subject of $y-2\sqrt{x-3}$, such as writing $\sqrt{y-2} = x-3 \Rightarrow x = \sqrt{y-2}+3$.

In part **(iii)** a small, but significant number, managed to lose the '+' sign in $2+\sqrt{(x-3)}$ at the very last stage and ended with $\frac{12}{2\sqrt{9}}+2=4$. A small number in part **(iii)** carried the negative value from the square root and offered –10 as a second solution.

Answers: **(i)** f(x) › 2; **(ii)** $(x-2)^2+3$; **(iii)** 4.4.

Question 7

Answers to this question ranged from those with complete accuracy to those with muddled ideas on combining logarithms. Those in between tended to gain full marks on the latter parts while failing on the first two although a few reversed this trend.

Mistakes made by some candidates included writing $\log\sqrt{X} = \sqrt{\log X} = \sqrt{9} = 3$ for part **(i)**, $\log\left(\frac{1}{x}\right) = \frac{1}{\log X} = \frac{1}{9}$ for part **(ii)** and $\log(XY) = (\log X)(\log Y) = 9 \times 6 = 54$ in part **(iii)**. Not all candidates could evaluate log 1 and some were unable to cope with the change of base in part **(iv)**.

Answers: **(i)** 4.5; **(ii)** –9; **(iii)** 15; **(iv)** 1.5.

Question 8

Part **(i)** was often fudged or omitted altogether. A common tactic was to use the given answer for the area to find *PQ* and then to use this value of *PQ* to find the area forming a circular argument. As the question asked to find *PQ*, even correct solutions were often overlong due to the use of the distance formula although some did realise that the *y*-coordinate of *P* was all that was needed. In part **(ii)** most were able to find *t* correctly, although some equated *A* to zero. There were few errors in part **(iii)** but many omissions; an alarming number of candidates concentrated so much on proving the nature that they missed the marks allocated to finding the value of *A*.

Answers: **(ii)** 3; **(iii)** 108, maximum.

Question 9

Candidates often gained full marks or zero marks for this question. For those who gained nothing it was often very difficult to follow the method used as almost no statements or explanations were given. Some candidates were unsure whether to use permutations or combinations, so compromised by including some of each. The results from this question did not follow the general trend in that some of the good candidates failed to make any genuine progress after part **(i)** yet quite a number of the weaker candidates were able to provide perfect solutions for all three parts.

Part **(i)** was well answered by most candidates although a small number insisted on using a permutations approach rather than a combinations approach.

Part **(ii)** was often done well but many candidates were unsure whether to add or multiply.

Part **(iii)** caused the most problems with a variety of methods attempted. The most successful were those who added the combinations for the three allowed combinations. Those who tried to subtract disallowed combinations from their answer to the first part gave themselves more work and usually overlooked the fact that some combinations were discounted on more than one occasion.

Answers: **(i)** 126; **(ii)** 36; **(iii)** 72.

Question 10

This was one of the most successful questions for many candidates. The substitution method usually led to the correct quadratic although those who substituted for *y* made more errors. The basic method at each stage was usually correct enabling candidates to gain high scores despite inaccuracies. The only significant common error was to use either *A* or *B* rather than the mid-point in the equation of the perpendicular bisector. Many also found the gradient of *AB* using the points rather than interpreting the equation of the line.

Answer: $x - 2y + 54 = 0$.

Question 11

This was another good question for most of the more able candidates who possibly only dropped odd marks through giving only one solution to one or more parts. Common errors were the misuse of Pythagoras's Theorem in the first two parts.

Part **(a)(i)** was usually very well done with most candidates managing to form the equation $\tan x = 1.5$ although a few, having got 56.3° then managed to get an incorrect second angle whilst some managed to get wrong excess answers by first factorising as $\cos x(2 \tan x - 3) = 0$.

Part **(a)(ii)** was often well done with many completely correct answers. Errors included factorising wrongly as $(\cos y - 2)(2\cos y + 1)$.

Part **(b)** was not quite as successfully answered as part **(a)** with a number of candidates using degrees or thinking that $\sin(2z + 1)$ could be 'expanded' as $\sin 2z + \sin 1$. Others also lost out on full marks by using premature approximation in their value for $2z + 1$.

Answers: **(a)(i)** 56.3°, 236.3°; **(ii)** 60°, 300°; **(iii)** 0.0599, 0.511.

Question 12 EITHER

This was the more popular alternative, although quite a few candidates made no, or virtually no, attempt at either. Many candidates gained full marks for this question but for others the question was found to be difficult, because there was some uncertainty about how to integrate an exponential function. It was common to misread the instruction 'in terms of e' in both parts, and it was also common to find the *y*-coordinate of *R* calculated, although it was not required. In part **(i)** many candidates failed to include the constant of integration, which meant that the *y*-coordinate of *Q* was incorrect. In part **(ii)** many non-linear tangents with gradients such as $e^{\frac{x}{2}}$ were used in an attempt to find *R* and many left their final answer as $\frac{2}{e^1 - e^0}$ or $\frac{2}{e^1 - 1}$ instead of $\frac{2}{e - 1}$.

Answers: **(i)** $2e + 3$; **(ii)** $\frac{2}{e - 1}$.

Question 12 OR

There were a number of completely correct solutions but also many with major errors. Differentiation of an exponential function seemed to work better than integration and the normal gradient was usually applied. Some candidates however tried to substitute directly into the given equation.

Point *A* was often given as (0, 5). For some reason the coordinates of *B* often included a non-zero *y*-value.

Many candidates knew to integrate to find the area but the $e^{\frac{x}{2}}$ became expressions such as $\frac{1}{2}e^{\frac{x}{2}}$ or $\frac{e^{\frac{x}{2}+1}}{\frac{x}{2}+1}$.

A number of candidates used limits of x_B and y_C in the integral rather than 0 and x_B, while others failed to take into account the area of rectangle *OBCD*.

Some candidates attempted to find the area between the curve and the *y*-axis but the integral of the natural logarithm defeated all but a few exceptional candidates.

Answers: **(i)** (3, 0); **(ii)** $e^{1.5} + 2$ or 6.48.